



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2026

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 Find the roots of the equation $w^5 + 32 = 0$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



[5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



3 Let $I_n = \int_0^a (\ln(x+1))^n dx$, where n is a non-negative integer and $a > 0$.

(a) By considering the derivative of $(x+1)(\ln(x+1))^n$, or otherwise, show that for $n \geq 1$

$$I_n = (a+1)(\ln(a+1))^n - nI_{n-1}.$$

[3]





(b) Find I_2 in terms of a .

[3]





4 Find the particular solution of the differential equation

$$49\frac{d^2x}{dt^2} + 14\frac{dx}{dt} + x = 196e^{\frac{1}{7}t},$$

given that, when $t = 0$, $x = 7$ and $\frac{dx}{dt} = 0$. [10]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



7

[illegible]

- 5 (a) Starting from the definitions of sech and $\tanh$ in terms of exponentials, prove that

$$1 - \operatorname{sech}^2 t = \tanh^2 t.$$

[3]

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It is given that

$$x = t \sinh t \quad \text{and} \quad y = t \cosh t.$$

- (b) Show that $\frac{dy}{dx} = \frac{t \tanh t + 1}{t + \tanh t}$.

[3]

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(c) Show that $\frac{d^2y}{dx^2} = \frac{(t^2 - 2)\operatorname{sech}^n t}{(t + \tanh t)^n}$, where n is an integer to be determined.

[6]





6 (a) Show that an appropriate integrating factor for

$$(1-x^2)\frac{dy}{dx} + y = (1-x^2)\sqrt{1-x}$$

is $\sqrt{\frac{1+x}{1-x}}$.

[4]

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(b) Hence find the solution of the differential equation

$$(1-x^2)\frac{dy}{dx} + y = (1-x^2)\sqrt{1-x}$$

for which $y = 1$ when $x = 0$.

[6]



- 7 (a) Use the substitution $u = 1 - x^2$ to find $\int \frac{x}{\sqrt{1-x^2}} dx$. [3]

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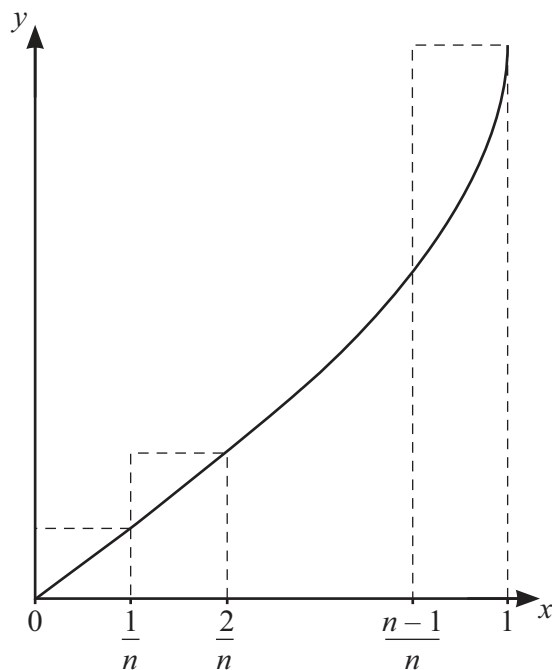
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The diagram shows the curve with equation for $y = \sin^{-1}x$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{n} \sin^{-1}\left(\frac{r}{n}\right) \geq \frac{1}{2}\pi - 1. \quad [5]$$

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- (c) Use a similar method to find, in terms of n , an upper bound for $\sum_{r=1}^n \frac{1}{n} \sin^{-1}\left(\frac{r}{n}\right)$. [3]

- (d) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \sin^{-1}\left(\frac{r}{n}\right)$. [1]



- 8 (a) The constants a and b are such that the system of equations

$$6x - 12y - 12z = 1,$$

$$ax + 4y + z = 2,$$

$$2x + by - 8z = 3,$$

does not have a unique solution. Find a in terms of b .

[3]

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- (b) Given that $a = -\frac{1}{4}$ and $b = -32$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically.

[3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -12 & -12 \\ -1 & 4 & 1 \\ 2 & -8 & -8 \end{pmatrix},$$

- (c) Show that $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and state the corresponding eigenvalue. [2]

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- (d) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$. [7]

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Additional page

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